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Speed and shape of dust acoustic solitary waves in a three-component dusty plasma with vortex-like ion distribution

Prasanta Chatterjee*[1] and Bholanath Sen[2]

[1] Department of Mathematics, Siksha Bhavana, Visva Bharati, Santiniketan-731 235, West Bengal, India

[2] Chhatna Chandidash Vidyapith, Chhatna-722 132, Bankura, West Bengal, India

E-mail prasantachatterjee1@rediffmail.com

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Abstract : The existence of dust acoustic solitary waves are investigated on the non-linear, unmagnetized homogeneous three-component dusty plasma whose components are inertial charged dust particles, Boltzmann distributed electrons and vortex like distributed ions. The Sagdeev's pseudopotential technique is employed for this study. Pseudopotential is determined in terms of u_d, the dust ion speed. It is found that there exists a critical value of u_{dt}, the value of u_d at which $u_d'^2 = 0$ and beyond which the solitary waves cease to exist.

Keywords Dust acoustic solitary waves, speed, shape, pseudopotential technique

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1. Introduction

There has been a great deal of interest in the study of different types of collective processes in dusty plasma. It has importance in space and astro-physical plasma, laboratory and environment. The role of dusty plasmas in cometary tails, asteroid zones, planetary ring, intersteller medium, earth's ionosphere and magnetosphere has been studied by several authors [1-7]. It also plays a vital role in understanding different types of new and interesting aspects in other fields like low temperature physics, radio frequency plasma discharge [8], coating and etching of thin films [9], plasma crystal [10,11] *etc*. The waves in dusty plasma are studied in distinct modes like dust-acoustic mode (DA) [12-14], dust ion-acoustic mode (DIA) [15,16], dust Berstain-Greene-Kruskal (DBGK) mode [17], dust lattice (DL) mode [18], Shukla-Verma mode [19], dust-drift mode [20] *etc*. Recently, a number of theoretical studies of DIA soliton [21, 22], DA soliton [23, 24] and DL soliton [25] are done with low frequency dust associated electrostatic and electromagnetic waves. The DIA solitary and shock waves and DI solitary waves have also been investigated experimentally [13, 26]. To study soliton solution, most of the authors how-ever used reductive perturbative technique (RPT) and obtained Korteweg-de-Vries (KdV) or KdV type equations [27-29].

A few years ago, Malfliet and Wieers [30] reviewed the studies on solitary waves and found that RPT is based on the smallness of amplitude. So to study large amplitude solitary waves one has to employ some non-perturbative techniques. Sagdeev's pseudopotential technique [31] is one such techniques. More recently, Johnston and Epstein [32] derived the Sagdeev's potential in terms of u, the ion-acoustic speed instead of ϕ, the electric potential. They observed that a very small change in initial conditions destroys the oscillatory behaviour of the solitary waves. Chatterjee and Das [33] observed the effect of electron inertia on the critical value for which the oscillatory behaviour of the large amplitude solitary waves are destroyed. Maitra and Roychoudhury [34] also studied large amplitude, dust acoustic solitary waves by a similar analysis.

But the ion and electron distribution functions are significantly modified in the presence of large amplitude solitary waves which are excited by the two-stream instability [35]. In

* Corresponding Author

this context, one may consider a vortex-like distribution [36,37] for ions in plasma. Accordingly, here we consider the trapped or vortex like ion distribution $f_i = f_{if} + f_{it}$, where

$$f_{if} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(v_i^2+2\phi)} \quad \text{for } |v_i| > \sqrt{-2\phi} \tag{1}$$

and
$$f_{it} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\sigma_{it}(v_i^2+2\phi)} \quad \text{for } |v_i| \le \sqrt{-2\phi} . \tag{2}$$

The ion distribution function is continuous in velocity space and satisfies the regularity requirements for an admissible BGK solution [17]. Here, the ion velocity v_i in eqs. (1) and (2) is normalized by the ion thermal speed v_{Ti} ; $\sigma_{it} = T_i/T_{it}$, where T_i is the free ion temperature and T_{it} is the trapped ion temperature. Integrating the ion distributions over velocity space, we get ion number density as

$$n_i = I(-\phi) + \frac{1}{\sqrt{\sigma_{it}}} e^{-\sigma_{it}\phi} erf\left(\sqrt{-\sigma_{it}\phi}\right) \tag{3}$$

for $\sigma_{it} > 0$ and

$$n_i = I(-\phi) + \frac{1}{\sqrt{\pi|\sigma_{it}|}} W_D\left(\sqrt{-\sigma_{it}\phi}\right) \quad \text{for } \sigma_{it} < 0 , \tag{4}$$

where

$$I(x) = \left[1 - erf\left(\sqrt{x}\right)\right] e^{x} , \tag{5}$$

$$erf\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy , \tag{6}$$

$$W_D(x) = e^{-x^2} \int_0^x e^{y^2} dy . \tag{7}$$

In this paper, our aim is to study the existence of dust acoustic solitary waves by a similar analysis done in [24, 33] under the influence of vortex like distributed ions [36, 37].

The organization of the paper is as follows. In Section 2, basic equations are written for unmagnetized homogeneous, three-component dusty plasma. The governing second order ODE is derived. Condition for existence of soliton solution and results are discussed in Section 3. Section 4 is kept for conclusion.

2. Basic equations

We consider a three-component dusty plasma system consisting of extremely massive, micron sized, negatively charged, inertial dust grains, Boltzmann distributed electron and vortex-like ion distribution. Our analysis is based on the continuity and momentum fluid equation for ions, electrons and Poisson equation which are given below. The basic equations are

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0, \tag{8}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{\partial \phi}{\partial x} \tag{9}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_d - n_i + n_e , \tag{10}$$

where we take

$$n_i = e^{-\phi} erfc\sqrt{-\phi} + \frac{1}{\sigma_{it}} e^{-\sigma_{it}\phi} erf\sqrt{-\sigma_{it}\phi} , \tag{11}$$

and

$$n_e = \mu_0 e^{\alpha\phi} \tag{12}$$

Here, n_i is the ion number density normalized by n_{i0}, n_e is the electron number density normalized by n_{e0} and n_d is the dust particle number density normalized by n_{d0}, u_d is the dust fluid velocity normalized by the dust acoustic speed $C_d = (Z_d T_i/m_d)^{\frac{1}{2}}$, with T_i being the ion temperature, m_d being the mass of negatively charged dust particulates; ϕ is the electrostatic potential normalized by T_i/e , e being the magnitude of the electron change. The time and space variables are in the units of the dust plasma period $\omega_{pd} = \left(4\pi Z_d^2 n_{d0} e^2/m_d\right)^{\frac{1}{2}}$ and the Debye length $\lambda_{Dd} = \left(T_i/4\pi Z_d n_{d0} e^2\right)^{\frac{1}{2}}$, respectively. μ_1, μ_0 and α are defined as $\mu_1 = \frac{1}{1-\beta}$, $\mu_0 = \frac{\beta}{1-\beta}$, $\beta = \frac{n_{e0}}{n_{i0}}$ and $\alpha = T_i/T_e$. T_e is the electron temperature in units of Boltzmann constant.

In order to investigate the properties of the solitary wave solutions of equations (8)-(10), we assume that all dependent variables depend on a single independent variable $\xi = x - vt$ where v is the solitary wave velocity. The variable ξ is the special coordinate system moving with the solitary wave velocity, *i.e.* the wave frame. By substitution, $\frac{\partial}{\partial x} = \frac{d}{d\xi}$ and $\frac{\partial}{\partial t} = -v\frac{d}{d\xi}$ eqs. (8)-(10) reduce to

$$-v\frac{dn_d}{d\xi} + \frac{d(n_d u_d)}{d\xi} = 0 , \tag{13}$$

$$-v\frac{du_d}{d\xi} + u_d \frac{du_d}{d\xi} = \frac{d\phi}{d\xi} , \tag{14}$$

$$\frac{d^2\phi}{d\xi^2} = n_d - n_i + n_e . \tag{15}$$

To solve the above set of equations, the following boundary conditions are used : $\phi, \frac{d\phi}{d\xi} \rightarrow 0, n_d \rightarrow 1, u_d \rightarrow u_{d0}$ as $|\xi| \rightarrow \infty$.

Integrating eq. (13) and using the boundary conditions given above, we get

$$n_d = \frac{\cdot}{v - u_d} \tag{16}$$

Again from eq. (14) and using the boundary conditions, we get

$$) = -vu_d + u_d^{\bar{}} \tag{17}$$

Therefore after some calculations, we get

$$\frac{1}{\sqrt{1+\frac{2\phi}{v^2}}} \tag{18}$$

Using the last three eqs. (16-18) in eq. (15), we get

$$\frac{\partial^2 u_d}{\partial \xi^2} = \frac{\partial \psi(u_d)}{\partial u_d}, \tag{19}$$

where

$$\psi(u_d) = \frac{\psi_i(u_d) + \psi_e(u_d) + \psi_d(u_d)}{(v - u_d)^2} \tag{20}$$

and

$$\psi_d(u_d) = vu_d. \tag{21}$$

$$\psi_i(u_d) = -| \quad ^{vu_d - u_d^2/2} erfc\left(vu_d - \frac{u_d^2}{2}\right)$$

$$+ \frac{1}{\sigma_{it}\sqrt{\pi}} e^{vu_d - u_d^2/2} erf\left(vu_d - \frac{u_d^2}{2}\right)$$

$$\cdot \frac{1}{\sigma_{it}\sqrt{\pi}} \sqrt{vu_d - \frac{u_d^2}{2}(\sigma_{it} - 1)} + 1, \tag{22}$$

$$\psi_e(u) = \frac{\mu_0}{\alpha}\left(1 - e^{\alpha(-vu_d + u_d^2/2)}\right). \tag{23}$$

Expanding *erf* and *erfc* functions and neglecting much higher order terms $Q(\phi^4)$, eq. (18) can be written as

$$\psi_i(u) = -\mu_1 \; 1 + vu_d - \frac{u_d^2}{2} - \frac{4(1-\sigma_{it})}{3\sqrt{\pi}}\left(vu_d - \frac{u_d^2}{2}\right)^{\frac{\cdot}{2}}$$

$$-vu_d + \cdot \quad \frac{8(1-\sigma_{it}^2)}{15\sqrt{\pi}} | \; vu_d - \frac{u_d^2}{2} |^{\frac{5}{2}} + \cdot \; \left| -vu_d + \frac{u_d^2}{2} \right.$$

$$\frac{16(1-\sigma_{it}^2)}{105\sqrt{\pi}}\left(vu_d - \frac{u_d^2}{2}\right) + \frac{\cdot vu_d + \cdot \frac{u_d^2}{})^4}{24} \tag{24}$$

Hence, $\psi(u)$ and $d^2u/d\xi^2$ can be obtained up to $O(\phi^4)$ from eqs. (19-24). One can also write

$$\psi(u_d) = \frac{(u_d')^2}{2} \tag{25}$$

3. Soliton solution, results and discussion

To find the region of existence of solitary waves, one has to study the nature of the functions $\psi(u_d)$ and $\phi_1(u_d)$ defined by

$$u_d'' = \frac{\partial \psi_d}{\partial u_d} = \phi_1(u_d). \tag{26}$$

For solitary wave (see Refs. [32-34], $\phi_1(u_d)$ will have two roots, one being at $u_d = 0$ and other at some point $u_d = u_{d1} (\geq 0)$. Also $\phi_1(u_d)$ should be positive on the interval $(0, u_{d1})$ and negative on (u_{d1}, u_{dmax}), where u_{dmax} is obtained from the nonzero root of $\psi_d(u_d)$. To get the shape of the travelling solitary wave one has to solve $\phi_1(u_d) = u_d''$ numerically with suitable boundary condition.

Figure 1 shows the plot of $\psi_d(u_d)$ *vs* u_d for $v = 1.08$. Other parameters are $\alpha = 0.05$, $\beta = 0.05$, $\sigma_{it} = 1$; and therefore, $\mu_0 = 0.05263$ and $\mu_1 = 1.05263$. It is seen that $\psi_d(u_d)$ crosses the

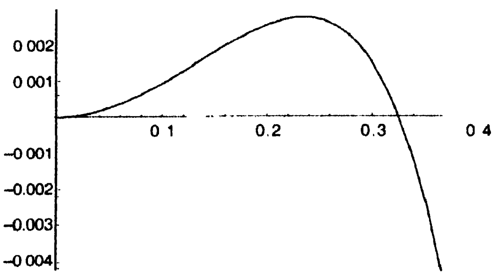

Figure 1. The plot of $\psi(u_d)$ *vs* u_d for $v = 1.1$. Other parameters are $\alpha = 0.05$, $\beta = 0.05$, $\sigma_{it} = 1$; and therefore, $\mu_0 = 0.05263$ and $\mu_1 = 1.05263$. (Read ψ in place of y and ξ in place of x in all figures).

u_d axis at $u_d = u_{d1} = 0.324728$. Hence, the amplitude of the solitary wave for this set of parameters will be 0.324728.

To get the shape of the solitary wave, we have solved numerically $u_d'' = \phi_1(u_d)$ with $u_d = 0.324728$, $u_d'' = 0$ and Figure 2 depicts the soliton solution $u_d(\xi)$ plotted against ξ. Other

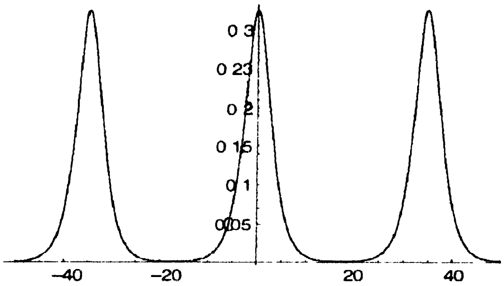

Figure 2. The soliton solution $u_d''(\xi)$ plotted against ξ for u_{d1} = 0 324728 Other parameters are same as those in Figure 1

parameters are same as those in Figure 1. It is seen that u_{d1} = 0.324728 is the critical value for u_d. For $u_d > u_{d1}$ the soliton solution ceases to exist and it is shown in Figure 3. In this figure, u_{d1} is taken as 0.324729 (all the other parameters are same

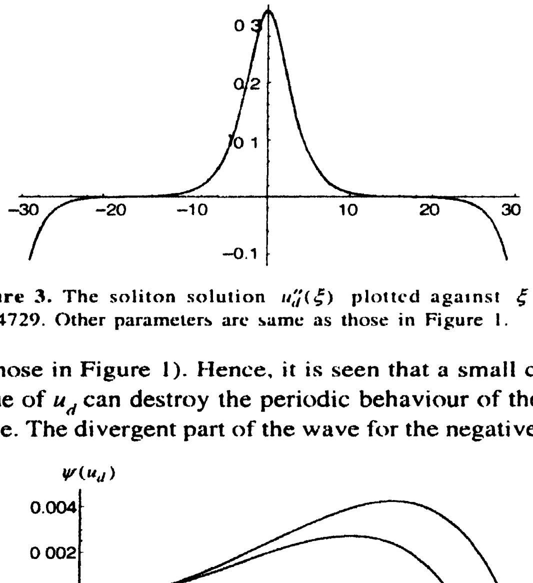

Figure 3. The soliton solution $u_d''(\xi)$ plotted against ξ for u_{d1} = 0.324729. Other parameters are same as those in Figure 1.

as those in Figure 1). Hence, it is seen that a small change of value of u_d can destroy the periodic behaviour of the solitary wave. The divergent part of the wave for the negative value of

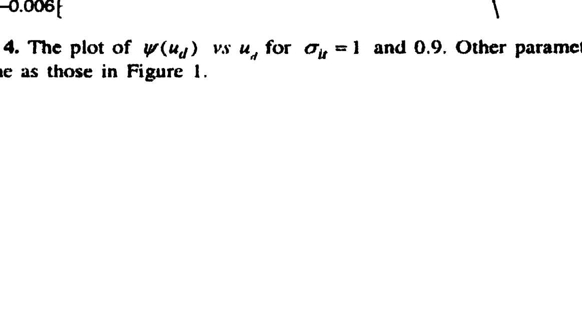

Figure 4. The plot of $\psi(u_d)$ *vs* u_d for $\sigma_{it} = 1$ and 0.9. Other parameters are same as those in Figure 1.

u_d can not be shown because of the presence of the square root of u_d in the differential equation. In Figure 4, $\psi(u_d)$ is plotted against u_d for different values of σ_{it}, *viz.* σ_{it} = 1, 9. It is seen that the amplitude of the solitary waves increases with the decrease of σ_{it}. Hence, it is seen that σ_{it} plays a significant role in forming and breaking the solitary waves in plasma.

4. Conclusion

The pseudopotential method is employed to study the speed and shape of the dust acoustic solitary waves in a three-component dusty plasmas. Vortex-like ion distribution is considered for ions and Boltzmann distribution is considered for electrons. Sagdeev's potential is obtained in terms of u_d, the dust fluid velocity. It is seen that there exists a critical value of u_d, at which $u_d'^2 = 0$, beyond which the soliton solution would not exist. This technique can be extended to the study of the existence of the dust acoustic solitary waves with charge fluctuation, non-thermal distribution of electrons *etc*. Work is in progress in these directions.

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